A Credibility Approach for Fuzzy Stochastic Data Envelopment Analysis (FSDEA)

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Abstract. It is well known that Data Envelopment Analysis (DEA) is a relative efficiency measurement tool, which uses optimization techniques to automatically calculate the weights assigned to the crisp deterministic multiple inputs and outputs of a set of the Decision Making Units (DMUs) being assessed. However, crisp deterministic data requirement delimits an application to the real world problems where some input or output measures likely are based on the value judgment of the decision makers. In this paper, the Fuzzy Stochastic Data Envelopment Analysis (FSDEA) model in the case of trapezoidal fuzzy numbers distributed with normal distribution in dual term is proposed, which is solved by two steps of transformation. First, the fuzzy deterministic DEA (FDDEA) model is converted by the concept of chance-constrained programming. Next, the credibility approach is used to convert a FDDEA model into a well-defined credibility programming model, in which fuzzy variables are replaced by expected credits.

Keywords: Chance constrained programming, Credibility measure, Data envelopment analysis, Fuzzy mathematical programming, Performance evaluation, Possibility Theory

1. INTRODUCTION

The traditional DEA is a widely applied non-parametric mathematical programming technique in the sense that no choice of a parametric functional form is needed in the estimation of the frontier for measuring and benchmarking the relative efficiency of homogenous decision making units (DMUs) with multiple inputs and multiple outputs. (Charnes et al. 1978, Cooper et al. 2000, Zhu 2003) This technique compares an evaluated (target) DMU or DMU, with other DMUs that utilize the same multiple inputs to produce the same multiple outputs based on operations research techniques to automatically calculate the weights assigned to inputs and outputs of each DMU. Numerous research papers on efficiency measurement of real life problems using DEA have been conducted. For example, Chilingerian (1995) used DEA and a multi-factor Tobit analysis to study of the clinical efficiency of 36 physicians in a single hospital. Sueyoshi (1995) proposed a new application of DEA for comparing performances in different time period. Nash and Sterna-Karwa (1996) applied DEA to measure 4 financial products cross selling effectiveness at the bank branch level. Tofallist (1997) used DEA to evaluated 14 major international passenger carriers for the year 1990. Hong et al. (1999) used DEA to evaluate the efficiency of the system integration (SI) projects and suggested the methodology which overcomes the limitation of DEA through hybrid analysis utilizing DEA along with machine learning. Al-Shammari (1999) evaluated the operational efficiency of 55 manufacturing organizations in Jordan by modified DEA model. Zhu (2000) used DEA to identify the best practice of 500 companies which was analyzed by Fortune magazine in 1995. Martin and Román (2001) applied a DEA model to analyze the technical efficiency and performance of each individual Spanish airport.

The formulation used in this paper is based on the most basically model of DEA so called the CCR model initially introduced by Charnes et al. (1978) under the assumption of constant returns to scale (CRS) of the efficient production frontiers. There are two models available to estimate the efficient frontier. One is dealing with minimizing inputs while producing at least the given output level, which is called the input oriented CCR model (CCR-I). The other attempts to maximize outputs while using no more than the observed amount of any input, which is called the output oriented CCR model (CCR-O). In addition to the CCR model, other well-known DEA models, each of which are extensions of CCR model, include the BCC model, which is built on the assumption of variable returns to scale (VRS) of the efficient production frontiers by Banker et al. (1984). There are two BCC models, one is dealing with inputs (BCC-I) and the other is the output oriented BCC model (BCC-O). Charnes et al. (1985) developed the additive DEA model (ADD) which considered possible input decreases as well as output increases simultaneously in the same production possibility set as in the BCC and CCR model.

2. DATA ENVELOPMENT ANALYSIS

For the CCR model, suppose that there are n DMUs are evaluated, each of which consumes the same type of m inputs and produces the same type of s outputs for j = 1, ..., n DMUs. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive. If the input and output data for DMUj are (x_{ij}, ..., x_{mj}) and (y_{1j}, ..., y_{sj}) respectively, then the efficiency of a target DMU or DMU_o where o range over 1, ..., n is measured by solving the fractional programming problem (FP) to obtain value of input weights v_i and output weights u_r for i = 1, ..., m and r = 1, ..., s as decision variables.

\[ \text{(FP-I)} \quad \max \theta = \sum_{i=1}^{m} \frac{u_i y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \quad (1) \]

Subject to \[ \sum_{i=1}^{m} v_i x_{io} \leq 1 \text{ for } j = 1, ..., n \quad (2) \]

\[ u_r, v_i \geq 0 \quad (3) \]

In the case of input oriented, the FP model in (1)-(3) is replaced with a linear programming problem (LP) by limiting denominator of the objective function in (1) to 1 and moving it to a constraint. After multiplying both sides of (2) by denominator then the FP model is equivalent to the following LP model.

\[ \text{(CCR-I)} \quad \max \theta = \sum_{r=1}^{s} u_r y_{ro} \quad (4) \]

Subject to \[ \sum_{i=1}^{m} v_i x_{io} = 1 \quad (5) \]

\[ \sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{io} \leq 0 \text{ for } j = 1, ..., n \quad (6) \]

\[ u_r, v_i \geq 0 \quad (7) \]

From a primal problem of CCR-I in (4)-(7), which is called a multiplier model, the DMU_o is evaluated to be CCR-efficient if \( \theta^* = 1 \) and there exists at least one optimal \( (v_i^*, u_r^*) \) with \( v_i^* > 0 \) and \( u_r^* > 0 \). Otherwise, the DMU_o is CCR-inefficient. For every LP there is the dual problem, in which the roles of variables and constraints are reversed. Suppose that the dual problem of the CCR model is expressed with a real variable \( \theta \) and dual variables \( \lambda_j \) for \( j = 1, ..., n \), then the dual problem of CCR-I (DCCR-I) is the following LP model.

\[ \text{(DCCR-I)} \quad \min \theta \]

Subject to \[ \theta x_{io} - \sum_{j=1}^{n} \lambda_j x_{oj} \geq 0 \text{ for } i = 1, ..., m \quad (9) \]

\[ \sum_{j=1}^{n} \lambda_j y_{oj} - y_{ro} \geq 0 \text{ for } r = 1, ..., s \quad (10) \]

\[ \theta \text{ Unrestricted}, \lambda_j \geq 0 \quad (11) \]

From DCCR-I in (8)-(11), which is called an
envelopment model, the DMU \textsubscript{o} is determined to be radial or technical efficiency or CCR-inefficiency if and only if an optimal solution satisfies \( \theta^* = 1 \) and the DMU \textsubscript{o} is determined to be CCR-efficiency if and only if an optimal solution satisfies \( \theta^* = 1 \) and all slacks are zero. Therefore, DCCR-I can be extended as follows.

\[
\text{(DCCR-I)} \quad \min \theta - \varepsilon (\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \quad (12)
\]

Subject to
\[
\sum_{j=1}^{n} \lambda_{ij} x_{ij} - s_i^- = 0 \quad (13)
\]
\[
\sum_{j=1}^{n} \lambda_{rj} y_{rj} - y_{ro} - s_r^+ = 0 \quad (14)
\]

\( \theta \) Unrestricted, \( \varepsilon > 0, \lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0 \) \quad (15)

Where \((s_i^-, s_r^+)\) are slacks of the input oriented envelopment model and \(\varepsilon\) is formulated as a non-Archimedean infinitesimal, which is added in the objective function effectively allows the minimization over \(\theta\) to preempt the optimization involving all slacks. However, using a specific value for \(\varepsilon\) is difficult in practices (Cooper et al. 2000, Zhu 2003), therefore the two-stage or two-phase procedure is used. In the first stage, LP problem in (8)-(11) is solved to find optimal \(\theta^*\). Then the second stage is finding a solution that maximizes the sum of all slack with assurance that \(\theta^*\) will not be possible to improve any input or output with worsening some other inputs or outputs. The two-phase procedure for DCCR is in the following form:

(Phase I-DCCR-I) min \(\theta\) \quad (16)

(Phase II-DCCR-I) \max \sum_{i=1}^{m} p_i x_{io} \quad (17)

Subject to
\[
\sum_{j=1}^{n} \lambda_{ij} x_{ij} - s_i^- = 0 \quad (18)
\]
\[
\sum_{j=1}^{n} \lambda_{rj} y_{rj} - y_{ro} - s_r^+ = 0 \quad (19)
\]

\( \theta \) Unrestricted, \(\lambda_j \geq 0, s_i^- \geq 0, s_r^+ \geq 0 \) \quad (20)

An optimal solution \((\lambda_j^*, s_i^*, s_r^*)\) from (16)-(20) is called the max-slack solution and if the max-slack solution satisfies both of \(s_i^* = 0\) and \(s_r^* = 0\), then it is called zero slack. For an inefficient DMU\textsubscript{o}, the efficiency of \((x_{io}, y_{ro})\) for DMU\textsubscript{o} can be improved by projecting DMU\textsubscript{o} into its reference set \(E_o\) based on the max slack solution, which is defined by

\[
E_o = \{j / \lambda_j^* > 0\} \quad \text{for } j \in \{1, 2, ..., n\}. \quad (21)
\]

Therefore, the CCR-I projections of the envelopment model are the following formula:

\[
x_{io,\text{improved}} = \theta^* x_{io} - s_i^* \quad (22)
\]
\[
y_{ro,\text{improved}} = y_{ro} + s_r^*. \quad (23)
\]

In the case of output oriented, let input weights \(p_i\) and output weights \(q_r\) for \(i = 1, ..., m\) and \(r = 1, ..., s\) be decision variables. The following FP model will be solved for an evaluated DMU\textsubscript{o} for \(o\) range over \(1, ..., n\).

(Phase I-DCCR-O) min \(\eta\) \quad (31)

(Phase II-DCCR-O) \max \sum_{i=1}^{m} p_i x_{io} \quad (24)

Subject to
\[
\sum_{j=1}^{n} \lambda_{ij} x_{ij} = 1 \quad (25)
\]
\[
\sum_{j=1}^{n} p_i x_{ij} \leq 0 \quad (26)
\]

Similar to the CCR-I, the FP-O is equivalent to the following LP model.

(CCR-O) \max \eta = \sum_{i=1}^{m} p_i x_{io} \quad (27)

Subject to
\[
\sum_{r=1}^{s} q_r y_{ro} = 1 \quad (28)
\]
\[
\sum_{r=1}^{s} q_r y_{rj} \leq 0 \quad (29)
\]

\( p_i, q_r \geq 0 \) \quad (30)

The DMU\textsubscript{o} is evaluated to be CCR-efficient if \(\eta^* = 1\) and there exists at least one optimal \((p_i^*, q_r^*)\) with \(p_i^* > 0\) and \(q_r^* > 0\). Otherwise, DMU\textsubscript{o} is CCR-inefficient.

To build the envelopment model for the output oriented model (DCCR-O), let \(\eta\) and \(\mu_j\) for \(j = 1, ..., n\) be dual variables and \((\lambda_j^*, \mu^*)\) be slacks of the output oriented envelopment model. Then the two-phase procedure of the dual problem of DCCR-O is the following LP model.

(Phase I-DCCR-O) min \(\eta\) \quad (31)
subject to \( s_i^{+} = 0 \) and \( s_i^{-} = 0 \) for all optimal solutions. Let \( \rho_i \) and \( \tau_r \) be slacks which can be inserted into the inequality outside braces to achieve equality for (45)-(46) in CC-FSDCCR-I.

\[
(x_{io}^{\text{improved}} = x_{io} - t_i^{*}) \quad (37)
\]

\[
(y_{ro}^{\text{improved}} = \eta y_{ro} + t_r^{*}) \quad (38)
\]

There are three reasons for solving the DCCR. First, the number of DMUs \( n \) is larger than the number of inputs and outputs \( m + s \) and hence it takes more time and larger memory to solve CCR with \( n \) constraints than to solve DCCR with \( m + s \) constraints. Second, using CCR the inefficient DMU cannot be improved to improve activity because reference set and max slack solution cannot be found. Finally, the interpretations of DCCR are more straightforward than those of CCR. (Cooper et al. 2000) The envelopment model is focused in this paper and there is no different between input and output oriented envelopment model in LP structure, thus DCCR-O will be not shown in this paper.

3. FUZZY STOCHASTIC DATA ENVELOPMENT ANALYSIS

In the preceding section, the original DCCR-I models are discussed based upon crisp deterministic data requirements but DEA modeling in the real world is based on information which is both of fuzzily imprecise and probabilistically uncertain. Some of the mathematical modeling techniques that model the inherent both of randomness and vagueness of the system include fuzzy and stochastic programming. In this section, the fuzzy and stochastic input oriented CCR model (FSDCCR-I) is proposed as follows.

\[
(\text{Phase I- FSDCCR-I}) \quad \min \theta \quad (39)
\]

\[
(\text{Phase II- FSDCCR-I}) \quad \max \sum_{i=1}^{m} t_i^{-} + \sum_{r=1}^{s} t_r^{+} \quad (40)
\]

Subject to \( \theta \tilde{x}_{io} - \sum_{j=1}^{n} \mu_j \tilde{x}_{ij} - t_i^{-} = 0 \) \( \quad (41) \)

\[
\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} - \tilde{y}_{ro} - s_r^{+} = 0 \quad (42)
\]

\( \theta \text{ Unrestricted, } \lambda_j \geq 0, s_i^{-} \geq 0, s_r^{+} \geq 0 \) \( \quad (43) \)

Where “\( \tilde{\bullet} \)” are fuzzy random variables.

In this paper, the concept of chance-constrained programming (CC) which was introduced by Charnes and Cooper (1959) is used to model FSDCCR-I. CC is a kind of stochastic optimization approaches. It is suitable for solving optimization problems with random variables included in constraints and sometimes in the objective function as well. The constraints are guaranteed to be satisfied with a specified probability or confidence level at the optimal solution found. Subsequently, some researchers like Charnes and Cooper (1962), Sengupta (1972, 1982, 1987, 1990), Land et al. (1992), Olesen and Petersen (1995), Cooper et al. (1996), Cooper et al. (1998), Li (1998), Sueyoshi (2000), Cooper et al. (2002) and many others established some theoretical results in field of stochastic programming. Stancu-Minasian and Wets (1976) presented a review paper on stochastic programming with a single objective function. The FSDCCR-I model from (8)-(11) in CC programming (CC-FSDCCR-I) is formed by

\[
(\text{CC-FSDCCR-I}) \quad \min \theta \quad (44)
\]

Subject to \( \Pr \left\{ \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io} \leq 0 \right\} \geq 1 - \alpha_i \quad (45) \)

\[
\Pr \left\{ \tilde{y}_{ro} - \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \leq 0 \right\} \geq 1 - \beta_r \quad (46)
\]

\( \theta \text{ Unrestricted, } \lambda_j \geq 0 \) \( \quad (47) \)

where “\( \Pr \)” means probability, \( 1 - \alpha_i \) and \( 1 - \beta_r \) are specified probabilities. DMU\(_o\) from (44)-(47) is determined to be stochastic efficient if and only if an optimal solution satisfies \( \theta^* = 1 \) and all slacks are zero for all optimal solutions. Let \( p_i \) and \( \tau_r \) be slacks which can be inserted into the inequality outside braces to achieve equality for (45)-(46) in CC-FSDCCR-I.
ξ and ζ respectively represent fuzzy standard deviation of + rjb, ija, rob respectively represent fuzzy + ija, ija, θ respectively represent fuzzy deterministic equivalent of the fuzzy stochastic input data structure with random disturbances for j = 1, ..., n; i = 1, ..., m and r = 1, ..., s. As follows:

\[ \tilde{x}_{ij} = \bar{x}_{ij} + \Delta_{ij} \tilde{\zeta}_{io} + \tilde{\phi}_{ij} \tilde{\xi}_{io} \]

where \( \bar{x}_{ij} \), \( \tilde{\phi}_{ij} \), \( \tilde{\xi}_{io} \), \( \Delta_{ij} \) and \( \tilde{\zeta}_{io} \) respectively represent fuzzy mean of fuzzy inputs and outputs standard deviation of fuzzy inputs and outputs variables. \( \Delta_{ij} \), \( \tilde{\phi}_{ij} \), \( \tilde{\xi}_{io} \) and \( \tilde{\zeta}_{io} \) represent symmetric disturbance or error terms of fuzzy inputs and outputs. Generally, error structures \( \zeta_{io} \), \( \zeta_{ij} \), \( \xi_{io} \) and \( \xi_{ij} \) are assumed to be standard normal distribution N(0,1), then fuzzy expected value and fuzzy variance of inputs and outputs variables are respectively defined by

\[ \tilde{x}_{ij} = \bar{x}_{ij} + \Delta_{ij} \tilde{\zeta}_{io} + \tilde{\phi}_{ij} \tilde{\xi}_{io} \]

Zr = \[ \frac{\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io} - \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) - E(\tilde{x}_{io})}{\sqrt{[\theta \lambda]^T \text{Cov}_i [\theta \lambda]}} \] (55)

\[ \tilde{y}_{ro} - \sum_{j=1}^{m} \lambda_j \tilde{y}_{rj} - \sum_{j=1}^{r} \lambda_j E(\tilde{y}_{rj}) \leq (1 - \beta_i) + \tau_r \] (50)

\[ \sum_{j=1}^{m} s_i^j + s_r^j \leq 1 - \alpha_i \] (52)

\[ \sum_{j=1}^{m} s_i^j + s_r^j \leq 1 - \beta_i \] (53)

\[ \theta \text{Unrestricted, } \lambda_j \geq 0, s_i^j \geq 0, s_r^j \geq 0. \] (54)

From (52) and (53), the level of probability is indicated by the values of \( s_i^j \) and \( s_r^j \), respectively, permits a further decrease in \( \tilde{x}_{io} \) and increase in \( \tilde{y}_{ro} \) for any sample of observations without worsening any other inputs or outputs.

4. THE EQUIVALENT FUZZY DETERMINISTIC DCCR-I MODEL

In this section, the method to convert FSDCCR-I into fuzzy deterministic equivalent of the fuzzy stochastic model is shown. Assume that the fuzzy inputs and outputs are distributed with normal distribution. Suppose that \( z_i \) and \( z_r \) respectively are fuzzy inputs and outputs distributed with standard normal distribution.

\[ z_i = \frac{\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io} - \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) + \theta E(\tilde{x}_{io})}{\sqrt{[\theta \lambda]^T \text{Cov}_i [\theta \lambda]}} \] (55)

\[ z_r = \frac{\sum_{j=1}^{m} \lambda_j \tilde{y}_{rj} - \sum_{j=1}^{r} \lambda_j E(\tilde{y}_{rj})}{\sqrt{[1 \lambda]^T \text{Cov}_i [\lambda]}} \] (56)

where \( [\theta \lambda] = (\theta, \lambda_1, \lambda_2, ..., \lambda_n)^T \), \( [\lambda] = (1, \lambda_1, \lambda_2, ..., \lambda_n)^T \), Cov_i and Cov_r are \((m + 1) \times (m + 1)\) and \((s + 1) \times (s + 1)\) matrices, respectively, which indicates variance and covariance of fuzzy output and input random variables for the jth DMU. Therefore, (52) and (53) are normalized in following formula.

\[ \text{Pr}(z_i \leq \frac{-s_i^j - \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) + \theta E(\tilde{x}_{io})}{\sqrt{[\theta \lambda]^T \text{Cov}_i [\theta \lambda]}}) = 1 - \alpha_i \] (57)

\[ \text{Pr}(z_r \leq \frac{-s_i^j - \sum_{j=1}^{n} \lambda_j E(\tilde{y}_{rj})}{\sqrt{[1 \lambda]^T \text{Cov}_i [\lambda]}}) = 1 - \beta_r \] (58)

Reformulate (57) and (58) to be

\[ \sum_{j=1}^{n} \lambda_j E(\tilde{x}_{ij}) - \theta E(\tilde{x}_{io}) + s_i^j = \Phi^{-1}(\alpha_i) \sqrt{[\theta \lambda]^T \text{Cov}_i [\theta \lambda]} \] (59)

\[ E(\tilde{y}_{ro}) - \sum_{j=1}^{m} \lambda_j E(\tilde{y}_{rj}) + s_r^j = \Phi^{-1}(\beta_r) \sqrt{[1 \lambda]^T \text{Cov}_i [\lambda]} \] (60)

where \( \Phi \) represents the normal cumulative distribution function and \( \Phi^{-1} \) is its inverse. Since (40) and (41) are expressed by the expected value and the variance-covariance matrices Cov_i and Cov_r, which are formulated by quadratic term. Thus solving two-phase problem of FSCCR is a non-trivial task.

In this paper, the linearization approach to obtain a linear deterministic equivalent model is used (Cooper et al. 1996, Cooper et al. 1998, Li 1998). Consider the input-output data structure with random disturbances for j = 1, ..., n; i = 1, ..., m and r = 1, ..., s. As follows:

\[ \tilde{x}_{ij} = \tilde{x}_{ij} + \Delta_{ij} \tilde{\zeta}_{io} + \tilde{\phi}_{ij} \tilde{\xi}_{io} \] (61)
E(\tilde{x}_{ij}) = \tilde{x}_{io} and E(\tilde{y}_{ij}) = \tilde{y}_{ij} \quad (62)

E(\tilde{y}_{ro}) = \tilde{y}_{ro} and E(\tilde{y}_{ro}) = \tilde{y}_{ro} \quad (63)

Var(\tilde{x}_{io}) = \tilde{a}_{io}^2 and Var(\tilde{y}_{ij}) = \tilde{a}_{ij}^2 \quad (64)

Var(\tilde{y}_{ro}) = \tilde{b}_{ro}^2 and Var(\tilde{y}_{ro}) = \tilde{b}_{ro}^2 \quad (65)

Since variance and covariance matrices are

\text{Cov}_i = \begin{pmatrix} \tilde{a}_{io} & \tilde{a}_{io} & \cdots & \tilde{a}_{io} \\ \tilde{a}_{io} & \tilde{a}_{i1} & \cdots & \tilde{a}_{i1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{io} & \tilde{a}_{i1} & \cdots & \tilde{a}_{in} \end{pmatrix} \quad (66)

\text{Cov}_r = \begin{pmatrix} \tilde{b}_{ro} & \tilde{b}_{ro} & \cdots & \tilde{b}_{ro} \\ \tilde{b}_{ro} & \tilde{b}_{r1} & \cdots & \tilde{b}_{r1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{ro} & \tilde{b}_{r1} & \cdots & \tilde{b}_{rm} \end{pmatrix} \quad (67)

Therefore,

\sqrt{[\theta \lambda^T]^{T} \text{Cov}_i [\theta \lambda]} = \theta \tilde{a}_{io} + \sum_{j=1}^{n} \lambda_j \tilde{a}_{ij} \quad (68)

\sqrt{[\lambda^T]^{T} \text{Cov}_r [\lambda]} = \tilde{b}_{ro} + \sum_{j=1}^{m} \lambda_j \tilde{b}_{rj} \quad (69)

The equivalent fuzzy deterministic model of the two-phase program for dual CCR model (FDDCCR) in the term of fuzzy LP model is

(Phase I-FDDCCR-I) min \theta \quad (70)

(Phase II-FDDCCR-I) max \sum_{i=1}^{m} s_i + \sum_{r=1}^{n} s_r \quad (71)

Subject to

\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} - \theta \tilde{x}_{io} + s_i = \Phi^{-1}(\alpha_i) \left( \theta \tilde{a}_{io} + \sum_{j=1}^{n} \lambda_j \tilde{a}_{ij} \right) \quad (72)

\tilde{y}_{ro} - \sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} + s_r = \Phi^{-1}(\beta_r) \left( \tilde{b}_{ro} + \sum_{j=1}^{m} \lambda_j \tilde{b}_{rj} \right) \quad (73)

\theta \text{ Unrestricted, } \lambda_j \geq 0, \ s_i \geq 0, \ s_r \geq 0. \quad (74)

5. THE EQUIVALENT CRISP DETERMINISTIC DCCR-I MODEL

5.1 Possibility and Necessity Measures

Possibility theory in the context of the fuzzy set theory was introduced by Zadeh (1978) which is dealing with non-stochastic imprecision and vagueness. A good reference on possibility theory is Dubois and Prade (1980) and Zimmermann (1996). Possibility and necessity measures are summarized and adopted to solve fuzzy DEA by Lertworasiriruk et al. (2003a, 2003b). Suppose that (\Theta_i, P(\Theta_i), \pi_i) for i = 1, \ldots, n are a possibility space with \Theta_i being the nonempty set of interest, P(\Theta_i) is the collection of all subset of \Theta_i, and \pi_i is the possibility measure from P(\Theta_i) to [0, 1], then

\pi(\Theta_i) = 0 and \pi(\Theta_i) = 1, and \quad (75)

\pi(\Theta_i \setminus \Theta_i) = \sup \{ \pi(\Theta_i) \} with each \Lambda_i \in P(\Theta_i). \quad (76)

Let \Psi be a fuzzy variable as a real-valued function defined over \Theta_i, therefore the membership function of is given by

\mu_{\Psi}(s) = \sup \{ \pi(\Theta_i) \} \Psi(\Theta_i) = s, \forall s \in R. \quad (77)

Let (\Theta, P(\Theta), \pi) be a product possibility space such that \Theta = \Theta_1 \times \cdots \Theta_n then

\pi(A) = \min \{ \pi_1(A_1) / A = A_1 \times \cdots \times A_n, A_i \in P(\Theta_i) \}. \quad (78)

To compare fuzzy variables (Dubois and Prade, 1980), let \tilde{a}_1, \ldots, \tilde{a}_n be fuzzy variables and f_j : \mathbb{R}^n \rightarrow \mathbb{R} be a real-valued function for j = 1, \ldots, m. The possibility measure of fuzzy event is given by

\pi( f_j(\tilde{a}_1, \ldots, \tilde{a}_n) \leq m for j = 1, \ldots, m = \sup \{ \min \{ \mu_{f_j}(s_i) / f_j(s_1, \ldots, s_n) \leq 0; j = 1, \ldots, m \} \}. \quad (79)

The necessity measure (N) of fuzzy event is defined as the impossibility of opposite event. If event A and A' are opposite event in \Theta, then N(A) = 1 – \pi(A'). Therefore, the necessity measure of fuzzy event is given by

N( f_j(\tilde{a}_1, \ldots, \tilde{a}_n) \leq 0 for j = 1, \ldots, m = 1 – \sup \{ \min \{ \mu_{f_j}(s_i) / \exists j \in \{ 1, \ldots, m \}, f_j(s_1, \ldots, s_n) > 0 \} \}. \quad (80)

5.2 Expected Credit of Normal Convex Fuzzy Variables

The credibility measure (Cr) of fuzzy event is defined by Liu (2001), which is the average of its possibility and necessity measures, i.e.,
An expected credit operator of a fuzzy variable \( \tilde{\Psi} \) on possibility space is defined by Liu (2001) as
\[
E(\tilde{\Psi}) = \int_{-\infty}^{\infty} \text{Cr}(\tilde{\Psi} \geq t) dt - \int_{-\infty}^{0} \text{Cr}(\tilde{\Psi} \leq t) dt.
\]  

Consider a fuzzy variable \( \tilde{\phi} \) on a possibility space \((\Theta, P(\Theta), \pi)\), which is called a normal convex fuzzy variable (see Fig.1) if \( \tilde{\phi} \) satisfies the following properties.

(i) The fuzzy variable \( \tilde{\phi} \) is normal if \( \sup_{s \in \mathbb{R}} |\mu_{\tilde{\phi}}(s)| = 1 \).

(ii) The \( \alpha \)-level set of the fuzzy variable \( \tilde{\phi} \) is defined by the set of elements that belong to \( \tilde{\phi} \) with membership of at least \( \alpha \), i.e., \( \tilde{\phi}_\alpha = \{ s \in \mathbb{R} | |\mu_{\tilde{\phi}}(s)| \geq \alpha \} \).

(iii) The fuzzy variable \( \tilde{\phi} \) is convex if \( \mu_{\tilde{\phi}}(\lambda s_1 + (1-\lambda)s_2) \geq \min\{ \mu_{\tilde{\phi}}(s_1), \mu_{\tilde{\phi}}(s_2) \} \) for all \( s_1, s_2 \in \mathbb{R} \) and \( \lambda \in [0,1] \).

\[
\mu_{\tilde{\phi}}(s)
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{normal_convex_fuzzy_variable}
\caption{Normal convex fuzzy variable}
\end{figure}

In this paper, the fuzzy variable \( \tilde{\Psi} \) is assumed to be the normal convex fuzzy variable, thus the expected credit of \( \tilde{\Psi} \) can be derived as follows.
\[
E(\tilde{\Psi}) = \lim_{M \to \infty} \left( \int_{-\infty}^{+M} \text{Cr}(\tilde{\Psi} \geq t) dt - \int_{-\infty}^{0} \text{Cr}(\tilde{\Psi} \leq t) dt \right).
\]  

Based on the necessity and the credibility definitions, the expected credit of \( \tilde{\Psi} \) is defined by
\[
E(\tilde{\Psi}) = \frac{1}{2} \lim_{M \to \infty} \left( \int_{-\infty}^{+M} \text{Cr}(\tilde{\Psi} \geq t) dt - \int_{-\infty}^{0} \text{Cr}(\tilde{\Psi} \leq t) dt + \int_{-\infty}^{+M} \pi(\tilde{\Psi} \geq t) dt - \int_{-\infty}^{0} \pi(\tilde{\Psi} \leq t) dt \right).
\]  

Since \( \tilde{\Psi} \) is the fuzzy variable with a normal convex function, the value of \( \pi(\tilde{\Psi} < t) \) and \( \pi(\tilde{\Psi} > t) \) are very close to \( \pi(\tilde{\Psi} \leq t) \) and \( \pi(\tilde{\Psi} \geq t) \). Therefore \( \pi(\tilde{\Psi} < t) \) and \( \pi(\tilde{\Psi} > t) \) are approximated by \( \pi(\tilde{\Psi} \leq t) \) and \( \pi(\tilde{\Psi} \geq t) \), respectively. Let \( \tilde{\phi}_L \) and \( \tilde{\phi}_U \) denote lower and upper bound of \( \alpha \)-level set of \( \mu_{\tilde{\phi}} \). Then the expected credit of the normal convex fuzzy variable \( E(\tilde{\Psi}) \) is
\[
\frac{1}{2} \left[ \int_{\tilde{\phi}_L} \pi(\tilde{\Psi} \geq t) dt - \int_{\tilde{\phi}_U} \pi(\tilde{\Psi} \leq t) dt \right].
\]  

To obtain an explicit form of the expected credit, terms \( \pi(\tilde{\Psi} \geq t) \) and \( \pi(\tilde{\Psi} \leq t) \) in (85) are evaluated by

Upper bound: Max \( \alpha \), st. \( (\tilde{\Psi})_L \geq t \)

Lower bound: Max \( \alpha \), st. \( (\tilde{\Psi})_L \leq t \).

Note that (85)-(87) were proposed and proved by Lertworasiriruk et al. (2003b). The results also applied for fuzzy variable of the LR type (Dubois and Prade, 1980). The expected credits of normal convex fuzzy variables are representing fuzzy parameters in FSDEA models because mean and standard deviation of fuzzy input-output variables usually have this type of membership function.

5.3 Mathematical Modeling in Fuzzy Environment

Decision making in a fuzzy environment was first developed by Bellman and Zadeh (1970). An application of fuzzy mathematical programming problem was proposed by Zimmermann (1978, 1985, 1996) and showed that the solution obtained by fuzzy linear programming was always efficient. Subsequently, some researchers like Sengupta (1992), Luhandjula (1996, 2003, 2004), Liu and Iwamura (1998), Liu (1998, 2001), Lertworasiriruk et al. (2003a, 2003b), Punyangarm et al. (2006) and many others have established some theoretical results in the field of fuzzy programming. In this paper, the concept of credibility approach is adopted as an alternative way for solving FDDCCR model.

5.2.1 CDDCCR-I Model

In this subsection, the credibility approach is used to convert the FDDCCR-I model to be CDDCCR-I. This approach treats fuzzy uncertain in the objective function and constraints by taking expected credit operators. Let \( E(\bullet) \) be expected credit operator of fuzzy variable, then (72) and (73) become
\[
E \left[ \sum_{j=1}^{n} \beta_{ij} (\tilde{x}_{ij} - \Phi^{-1}(\alpha_i \tilde{a}_{ij})) + \theta (\hat{x}_{i} \Phi^{-1}(\alpha_i \tilde{a}_{i})) + s_i \right] = 0 \quad \text{(88)}
\]
\[
E \left[ \sum_{j=1}^{n} \gamma_{ij} (\tilde{y}_{ij} - \Phi^{-1}(\beta_i \tilde{b}_{ij})) + \eta (\hat{y}_{i} \Phi^{-1}(\beta_i \tilde{b}_{i})) + s_i \right] = 0, \quad \text{(89)}
\]
Since $E[a\tilde{X} + b\tilde{Y}] = aE[\tilde{X}] + bE[\tilde{Y}]$ for any real numbers $a$ and $b$ (Liu and Liu 2003, Lertworasirikul et al. 2003b), then the FDDCCR-I model is equivalent to the following crisp deterministic LP model.

(Phase I-CDDCCR-I) min $0$  

(Phase II-CDDCCR-I) max $m \sum s^-_i + \sum s^+_r$  

Subject to

\[
\sum_{j=1}^{n} \lambda_j (E[\tilde{X}_j] - \Phi^{-1}(\alpha_i)E[\tilde{a}_j]) \\
- \theta (E[\tilde{Y}_m] - \Phi^{-1}(\beta_r)E[\tilde{b}_m]) \\
- \sum_{j=1}^{n} \lambda_j (E[\tilde{X}_j] - \Phi^{-1}(\beta_r)E[\tilde{b}_m]) + s^- = 0 \\
E[\tilde{Y}_m] - \Phi^{-1}(\beta_r)E[\tilde{b}_m] = 0 \\
- \sum_{j=1}^{n} \lambda_j (E[\tilde{X}_j] - \Phi^{-1}(\beta_r)E[\tilde{b}_m]) + s^+ = 0 \\
0 \text{ Unrestricted, } \lambda_j \geq 0, s^- \geq 0, s^+ \geq 0
\]

(92)

The inefficient DMU$_o$ can be improved by projecting DMU$_o$ into its reference set ($E_o$), therefore CDDCCR-I projections are given by following formula.

\[
\tilde{x}_{io,\text{improved}} = \theta^* (E[\tilde{x}_m] - \Phi^{-1}(\alpha_i)E[\tilde{a}_m]) - s^-
\]

\[
\tilde{y}_{io,\text{improved}} = E[\tilde{y}_m] - \Phi^{-1}(\beta_r)E[\tilde{b}_m] + s^+
\]

(96)

(97)

5.2.2 CDDCCR-I Model with Trapezoidal Fuzzy Inputs and Outputs

In this subsection, all of input and output variables are assumed to be trapezoidal fuzzy numbers. Unlike the case of crisp inputs and outputs, fuzzy mean and standard deviation are computed by the fuzzy arithmetic (Zimmermann 1996, Klir et al. 1997) and expected credit of fuzzy mean and standard deviation are computed by solving (85)-(87). Let fuzzy numbers $\tilde{c}_i = ((\tilde{c}_i)_0, (\tilde{c}_i)_1, (\tilde{c}_i)_2, (\tilde{c}_i)_3)$ for $i = 1, \ldots, n$ be trapezoidal fuzzy numbers observed in a sample by which $((\tilde{c}_i)_0, (\tilde{c}_i)_1, (\tilde{c}_i)_2, (\tilde{c}_i)_3)$ be crisp data of lower and upper bounds of $\alpha$-level set at $\alpha = 0$ and $\alpha = 1$. And let $\mu_\xi(s)$ be membership functions of $\tilde{c}_i$ (see Fig. 2), which are defined by

\[
\mu_{\tilde{c}_i}(s) = \begin{cases} 
0 & \text{for } s < (\tilde{c}_i)_0 \text{ or } s > (\tilde{c}_i)_U \\
\frac{s - (\tilde{c}_i)_0}{(\tilde{c}_i)_L - (\tilde{c}_i)_0} & \text{for } (\tilde{c}_i)_L \leq s < (\tilde{c}_i)_U \\
1 & \text{for } (\tilde{c}_i)_L \leq s \leq (\tilde{c}_i)_U \\
\frac{(\tilde{c}_i)_U - s}{(\tilde{c}_i)_U - (\tilde{c}_i)_1} & \text{for } (\tilde{c}_i)_1 < s \leq (\tilde{c}_i)_U
\end{cases}
\]

(98)

Fig. 2: Trapezoidal fuzzy number and its membership function

Let $(\tilde{c}_i)_0$ and $(\tilde{c}_i)_U$ respectively be the lower and upper bounds of the $\alpha$-level set of fuzzy number $\tilde{c}_i$, then closed crisp interval of $\tilde{c}_i$ are defined by

\[
(1 - \alpha)(\tilde{c}_i)_L + \alpha(\tilde{c}_i)_U \leq s \leq \alpha(\tilde{c}_i)_L + (1 - \alpha)(\tilde{c}_i)_U
\]

(99)

Based on concepts of the fuzzy arithmetic, the fuzzy sample mean of $\tilde{c}_i$ can be calculated by

\[
(\tilde{\bar{c}}_i)_a^U = \frac{a}{n} \sum_{i=1}^{n} (\tilde{c}_i)_L + \frac{(1-a)}{n} \sum_{i=1}^{n} (\tilde{c}_i)_U = \alpha\tilde{c}_i^U + (1-\alpha)(\tilde{c}_i)^0_U
\]

(100)

\[
(\tilde{\bar{c}}_i)_a^L = \frac{(1-a)}{n} \sum_{i=1}^{n} (\tilde{c}_i)_L + \frac{a}{n} \sum_{i=1}^{n} (\tilde{c}_i)_U = (1-\alpha)(\tilde{c}_i)^0_L + \alpha(\tilde{c}_i)_L
\]

(101)
respectively. (86) and (87) are used to find the maximum $\alpha$,

$$\tilde{c}_0^U - \alpha(\tilde{c}_0^U - \tilde{c}_1^U) \geq t$$  \hspace{1cm} (102)

$$\tilde{c}_0^L - \alpha(\tilde{c}_0^L - \tilde{c}_1^L) \leq t$$  \hspace{1cm} (103)

Since $\tilde{c}_0^L \leq \tilde{c}_1^L$ and $\tilde{c}_0^U \geq \tilde{c}_1^U$, then

$$\frac{(\tilde{c}_0^L - t)}{(\tilde{c}_0^L - \tilde{c}_1^L)} \leq \frac{(\tilde{c}_0^U - t)}{(\tilde{c}_0^U - \tilde{c}_1^U)}$$  \hspace{1cm} (104)

Therefore,

$$\pi(\tilde{c} \geq t) = \frac{(\tilde{c}_0^U - t)}{(\tilde{c}_0^L - \tilde{c}_1^L)}$$  \hspace{1cm} (105)

$$\pi(\tilde{c} \leq t) = \frac{(\tilde{c}_0^L - t)}{(\tilde{c}_0^L - \tilde{c}_1^L)}$$  \hspace{1cm} (106)

(105) and (106) are integrated to be

$$\int_{\tilde{c}_0}^{\tilde{c}_0}(\tilde{c} \geq t)dt = \frac{(\tilde{c}_0^U - t)}{2}$$  \hspace{1cm} (107)

$$\int_{\tilde{c}_0}^{\tilde{c}_0}(\tilde{c} \leq t)dt = \frac{(\tilde{c}_0^L - t)}{2}$$  \hspace{1cm} (108)

Substitute (107) and (108) in (85), thus the expected credit of $\tilde{c}$ can be calculated as follows.

$$E(\tilde{c}) = \frac{\tilde{c}_0^U + (\tilde{c}_1^U + \tilde{c}_1^L + \tilde{c}_0^L)}{4}$$  \hspace{1cm} (109)

In a similar manner, the fuzzy variance of $\tilde{c}$ denote that $\tilde{s}^2$ which is calculated by

$$\tilde{s}_0^2 = \left(1 - \alpha\right)^2 \left(\frac{n}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^U - n(\tilde{c}_0^U)^2\right)\right) + \frac{2(\alpha)(1-\alpha)}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^U (\tilde{c}_0^U)^2 - n(\tilde{c}_0)^U (\tilde{c}_0^U)^2\right) + \frac{\alpha^2}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^U - n(\tilde{c}_0^U)^2\right)$$  \hspace{1cm} (110)

$$\tilde{s}_0^2 = \left(1 - \alpha\right)^2 \left(\frac{n}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^L - n(\tilde{c}_0^L)^2\right)\right) + \frac{2(\alpha)(1-\alpha)}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^L (\tilde{c}_0^L)^2 - n(\tilde{c}_0)^L (\tilde{c}_0^L)^2\right) + \frac{\alpha^2}{n-1} \left(\sum_{i=1}^{n} (\tilde{c}_i)^L - n(\tilde{c}_0^L)^2\right)$$  \hspace{1cm} (111)

Let $\tilde{s}_{\tilde{c}_0}^2$, $\tilde{s}_{\tilde{c}_0}^2$, $\tilde{s}_{\tilde{c}_0}^2$ and $\tilde{s}_{\tilde{c}_0}^2$ be crisp sample variances at $\alpha = 1$ and $\alpha = 0$ in lower and upper bounds, $\tilde{s}_{\tilde{c}_0}^2$ and $\tilde{s}_{\tilde{c}_0}^2$ are crisp sample covariance, then upper and lower bounds of the $\alpha$-level set of fuzzy standard deviation are respectively given by

$$\tilde{\tilde{s}}_{\tilde{c}_0}^2 = \sqrt{(1-\alpha)^2 \tilde{s}_{\tilde{c}_0}^2 + 2(\alpha)(1-\alpha)\tilde{s}_{\tilde{c}_0}^2 + \alpha^2 \tilde{s}_{\tilde{c}_0}^2}$$  \hspace{1cm} (112)

$$\tilde{\tilde{s}}_{\tilde{c}_0}^2 = \sqrt{(1-\alpha)^2 \tilde{s}_{\tilde{c}_0}^2 + 2(\alpha)(1-\alpha)\tilde{s}_{\tilde{c}_0}^2 + \alpha^2 \tilde{s}_{\tilde{c}_0}^2}.$$  \hspace{1cm} (113)

Now we consider a correlation of two random variables, which are defined by

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$  \hspace{1cm} for $-1 \leq \rho_{XY} \leq 1$$  \hspace{1cm} (114)

where X and Y are random variables, $\rho_{XY}$ is the correlation of X and Y, $\sigma_{XY}$ is the covariance of X and Y, and $\sigma_X$, $\sigma_Y$ are standard deviation of X and Y respectively. The range over $[-1, 1]$ means that two random variables is highest correlated when correlation $\rho_{XY}$ is equal to 1 or -1. Subsequently, if the relationship of X and Y is linear equation then $\sigma_{XY} = \sigma_X \sigma_Y$.

Since $(\tilde{c}_0)_L = (\tilde{c}_0)_U + \xi$ and $(\tilde{c}_0)_0 = (\tilde{c}_0)^U + \zeta$ where $\xi$ and $\zeta$ are constant spreads of fuzzy numbers are linear equation, then sample covariance in (112) and (113) are given by

$$\tilde{s}_{\tilde{c}_0}^2 = \tilde{s}_{\tilde{c}_0}^2$$  \hspace{1cm} (115)

$$\tilde{s}_{\tilde{c}_0}^2 = \tilde{s}_{\tilde{c}_0}^2$$  \hspace{1cm} (116)

To find the maximum of $\alpha$, (112) and (113) are reformulated and substituted in (86) and (87), respectively.

$$\tilde{s}_{\tilde{c}_0}^2 - \alpha(\tilde{s}_{\tilde{c}_0}^2) \geq t$$  \hspace{1cm} (117)

$$\tilde{s}_{\tilde{c}_0}^2 - \alpha(\tilde{s}_{\tilde{c}_0}^2) \leq t$$  \hspace{1cm} (118)

By integrating (117) and (118) and substituting these results in (85), the expected credit of $\tilde{s}$ is given by

$$E(\tilde{s}) = \frac{\tilde{s}_{\tilde{c}_0}^2 + \tilde{s}_{\tilde{c}_0}^2 + \tilde{s}_{\tilde{c}_0}^2}{4}.$$  \hspace{1cm} (119)

Since the expected credit of $\tilde{c}$ and $\tilde{s}$ in (109) and (119) are involved a crisp summation term of corner points of each trapezoidal fuzzy variable, therefore crisp calculation in sample means and standard deviation at corner points are used in the CDDCCR-I model.
Suppose that there are \( t_i \) and \( w_r \) observations for inputs and outputs from DMU\(_j\) for \( j = 1, \ldots, n \), respectively, i.e.,

\[
\tilde{x}_{ij} = \tilde{x}_{ij}, \ldots, \tilde{x}_{ij}, \quad \text{for } i = 1, \ldots, m; \quad k_i = 1, \ldots, t_i \quad \text{and} \quad \tilde{y}_{jl} = \tilde{y}_{jl}, \ldots, \tilde{y}_{jl}, \quad \text{for } r = 1, \ldots, s; \quad l = 1, \ldots, w_r.
\]

Then expected credits of \( \tilde{x}_{ij} \), \( \tilde{y}_{jl} \), \( \tilde{a}_{ij} \) and \( \tilde{b}_{ij} \) are respectively defined by

\[
E(\tilde{x}_{ij}) = \frac{1}{4}(\tilde{x}_{ij})_0^U + \frac{1}{4}(\tilde{x}_{ij})_1^U + \frac{1}{4}(\tilde{x}_{ij})_1^L + \frac{1}{4}(\tilde{x}_{ij})_0^L \quad (120)
\]

\[
E(\tilde{y}_{jl}) = \frac{1}{4}(\tilde{y}_{jl})_0^U + \frac{1}{4}(\tilde{y}_{jl})_1^U + \frac{1}{4}(\tilde{y}_{jl})_1^L + \frac{1}{4}(\tilde{y}_{jl})_0^L \quad (121)
\]

\[
E(\tilde{a}_{ij}) = \frac{1}{4}(\tilde{a}_{ij})_0^U + \frac{1}{4}(\tilde{a}_{ij})_1^U + \frac{1}{4}(\tilde{a}_{ij})_1^L + \frac{1}{4}(\tilde{a}_{ij})_0^L \quad (122)
\]

\[
E(\tilde{b}_{ij}) = \frac{1}{4}(\tilde{b}_{ij})_0^U + \frac{1}{4}(\tilde{b}_{ij})_1^U + \frac{1}{4}(\tilde{b}_{ij})_1^L + \frac{1}{4}(\tilde{b}_{ij})_0^L \quad (123)
\]

For the CDDCCR-I model with trapezoidal fuzzy inputs and outputs, the expected credit of fuzzy variables in the CDDCCR-I model are replaced by (120)-(123), therefore

(Phase I-CDDCCR-I) min \( \theta \)

(Phase II-CDDCCR-I) max \( \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \) (124)

Subject to

\[
\frac{1}{4} \sum_{j=1}^{n} \lambda_j \left( (\tilde{x}_{ij})_0^U + (\tilde{x}_{ij})_1^U + (\tilde{x}_{ij})_1^L + (\tilde{x}_{ij})_0^L \right) -

\Phi^{-1}(a_i)((\tilde{a}_{ij})_0^U + (\tilde{a}_{ij})_1^U + (\tilde{a}_{ij})_1^L + (\tilde{a}_{ij})_0^L) \right) = -\theta \quad (125)
\]

\[
\frac{1}{4} (\tilde{a}_{io})_0^U + (\tilde{a}_{io})_1^U + (\tilde{a}_{io})_1^L + (\tilde{a}_{io})_0^L \right) + \Phi^{-1}(a_i)(\tilde{a}_{io})_0^U + (\tilde{a}_{io})_1^U + (\tilde{a}_{io})_1^L + (\tilde{a}_{io})_0^L) + s_i^- = 0 \quad (126)
\]

\[
\frac{1}{4} (\tilde{y}_{ro})_0^U + (\tilde{y}_{ro})_1^U + (\tilde{y}_{ro})_1^L + (\tilde{y}_{ro})_0^L -

\Phi^{-1}(b_r)((\tilde{b}_{ro})_0^U + (\tilde{b}_{ro})_1^U + (\tilde{b}_{ro})_1^L + (\tilde{b}_{ro})_0^L) \right) = -\theta \quad (127)
\]

\[
\theta \text{ Unrestricted, } \lambda_j \geq 0, \quad s_i^- \geq 0, \quad s_r^+ \geq 0 \quad (128)
\]

From (96)-(97), CDDCCR-I projections are given by following formula.

\[
\tilde{x}_{io,\text{improved}} = \frac{1}{4} \left( (\tilde{x}_{io})_0^U + (\tilde{x}_{io})_1^U + (\tilde{x}_{io})_1^L + (\tilde{x}_{io})_0^L \right) + 

\Phi^{-1}(a_i)((\tilde{a}_{io})_0^U + (\tilde{a}_{io})_1^U + (\tilde{a}_{io})_1^L + (\tilde{a}_{io})_0^L) \right) - s_i^- \quad (129)
\]

\[
\tilde{y}_{ro,\text{improved}} = \frac{1}{4} \left( (\tilde{y}_{ro})_0^U + (\tilde{y}_{ro})_1^U + (\tilde{y}_{ro})_1^L + (\tilde{y}_{ro})_0^L \right) -

\Phi^{-1}(b_r)((\tilde{b}_{ro})_0^U + (\tilde{b}_{ro})_1^U + (\tilde{b}_{ro})_1^L + (\tilde{b}_{ro})_0^L) \right) + s_r^+ \quad (130)
\]

Since (120)-(123) are crisp values, then the CDDCCR-I model in (124)-(128) and CDDCCR-I projections (129) and (130) can be solved by standard LP.

6. CONCLUSION

Unlike the traditional DEA, FSDEA model is based decision on information which is both randomness and vagueness of the system. This paper has presented two steps of transformation. First, the FDDEA model is converted by the concept of chance-constrained programming based on inputs and output variable which are distributed in normal distribution. Next, the credibility approach is used to convert a FDDEA model into a well-defined credibility programming model, in which fuzzy variables are replaced by expected credits. The result of this procedure is shown for a special case in which fuzzy variable are trapezoidal fuzzy numbers and we could summarize that the fuzzy sample mean and standard deviation of inputs and outputs are trapezoidal fuzzy numbers. By the expected credit approach, expected credits of fuzzy sample mean and standard deviation are easily calculated by using the crisp corner points of fuzzy numbers. After replacing them, the standard LP can be used to solve the CDDCCR-I model and CDDCCR-I projections.
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